$\nabla \cdot \vec{V} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$

$\nabla \times \vec{V} = \begin{pmatrix} \partial V_z/\partial y - \partial V_y/\partial z \\ \partial V_x/\partial z - \partial V_z/\partial x \\ \partial V_y/\partial x - \partial V_x/\partial y \end{pmatrix}$

1 Electric force

System total charge : 0

$e = 1.602 \times 10^{-19}$ $[C]$

$q = n \cdot e$ $[C]$

$k_E := \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9$ $[Nm^2C^{-2}]$

$\epsilon_0 \approx 8.8 \times 10^{-12}$ $[C^2N^{-1}m^{-2}]$

electron : $-e$, proton : $+e$

$\vec{r} = \vec{r_1} - \vec{r_2}$

$\vec{F}_{12} = k_E \frac{q_1 q_2}{\|\vec{r}\|^2} \cdot \frac{\vec{r}}{\|\vec{r}\|}$

$\vec{F}_Q(\vec{r}) = Q\vec{E}(\vec{r})$

$\vec{F}^C_{\text{res}} = \sum_i \vec{F}^C_i$, $\vec{E}^C_{\text{res}}(\vec{r}) = \sum_i \vec{E}^C_i(\vec{r})$

1.1 Charge density

Linear : $Q = \int_x \lambda(x)\, dx$

Surface : $Q = \int_x \int_y \sigma(x, y)\, dx\, dy$

Volume : $Q = \int_x \int_y \int_z \rho(x, y, z)\, dx\, dy\, dz$

1.2 Electric field

Field lines : $+ \to -$, tangent to E and $\perp$ to surface of conductor.

Inside conductor (static) : $E = 0$

Charge on conductor distributes on the surface

$\vec{E}^C(\vec{r}) = k_E \frac{Q}{\|\vec{r} - \vec{r}_q\|^2} \cdot \frac{\vec{r} - \vec{r}_q}{\|\vec{r} - \vec{r}_q\|}$ $[NC^{-1}]$

1.3 Dipole

$-Q \xrightarrow{\vec{l}} +Q$

Moment : $\vec{p} = Q \cdot \vec{l}$

Torque : $\vec{\tau} = \vec{p} \times \vec{E}_{\text{uniform}}$

Potential energy : $U = -\vec{p} \cdot \vec{E}$

Field $\perp \frac{\vec{l}}{2}$: $\vec{E}(\vec{r}) = k_E \frac{\vec{p}}{r^3}$

2 Gauss

Volume V of surface S

α : angle between $\vec{E}$ and $d\vec{S}_\perp$

Flux through a surface :

$\Phi = \oint \vec{E}(\vec{r}) \cdot d\vec{S}$ $[Nm^2C^{-1}]$

$= \oint E(\vec{r}) \cdot ds \cdot \cos\alpha$

Flux does not depend on the form of the surface.

Flux of E over closed surface is equal to the enclosed charge :

$\oint_S \vec{E} \cdot d\vec{S} = 4\pi k_E \sum_{q \in V} q$

$= 4\pi k_E \int_V \rho(\vec{r})\, dV$

$\equiv \frac{1}{\epsilon_0} \int_V \rho(\vec{r})\, dV$

First Maxwell (Gauss's law), divergence of E at any point is equal to the charge density :

$\nabla \cdot \vec{E} = 4\pi k_E \rho \equiv \frac{\rho}{\epsilon}$

$\Phi_E \equiv \oint \vec{E}\, d\vec{S} = 4\pi k_E q \equiv \frac{q}{\epsilon_0}$

3 Electric potential

$\Delta U \equiv U_f - U_i = -W$

$W_{\text{ext}} = \Delta U$

Potential energy of point charge :

$U_Q(\vec{r}) = Q \cdot \phi(\vec{r})$ $[J]$

Potential of point charge :

$\phi(\vec{r}) = k_E \frac{q}{|\vec{r}|}$ $[V][JC^{-1}]$

$\vec{E} = -\nabla\phi$, $\vec{F} = -\nabla U$

Work : does not depend of the path, is 0 over closed loop

If $\nabla \times E = 0$ then E is a potential field

4 Magnetic force

Current : flow of charge $I = \frac{dQ}{dt}$ $[A]$ $[Cs^{-1}]$

Current density : ($\vec{v}$: charge velocity)

$\vec{j} = \rho\vec{v}$ $[Am^{-2}]$ $[Cs^{-1}m^{-2}]$

Current through wire (cross-section S) :

$I = j \cdot S$

Charge conservation : $\frac{\partial\rho}{\partial t} + \nabla \cdot \vec{j} = 0$

Permeability of free space :

$\mu_0 = 4\pi \times 10^{-7}$ $[TmA^{-1}]$

$k_M := \frac{\mu_0}{4\pi}$

Force on two parallel conductors, lengths L :

$F_A = 2\, k_M L \frac{I_1 I_2}{r}$

$F(\alpha, \beta) = F_{\text{max}} \cdot \sin\alpha \cdot \sin\beta$

$d\vec{F} = k_M \frac{I_1 I_2}{r^2} \left[dl_1 \times \left[dl_2 \times \frac{\vec{r}}{r}\right]\right]$

Magnetic field :

$d\vec{F} = I dl \times \vec{B}(\vec{r})$, $dF = IdlB\sin\theta$

$d\vec{B}_i(\vec{r}) = k_M \frac{I_i dl_i (\vec{r} - \vec{r}_i)}{\|\vec{r} - \vec{r}_i\|^3}$

$B = B_1 + \ldots + B_n$

Force (Lorentz) on single charged particle :

$\vec{F} = q(E + \vec{v} \times \vec{B})$

Torque : $\vec{\tau} = \vec{r} \times \vec{F}$, $\tau = rF\sin\theta$

Ohm's law : $V = RI$ $[V]$, R : $[\Omega]$

Electromotive force : emf ε $[V]$

$\oint_{\partial\Sigma} \vec{E} dl = -\frac{d}{dt} \int_\Sigma \vec{B} d\vec{S}$

Third Maxwell (Faraday's law) : $\nabla \times E = -\frac{\partial\vec{B}}{\partial t}$

$\varepsilon \equiv \oint \vec{E}\, dl = -\frac{d\vec{\Phi}_B}{dt}$

4.1 Magnetic dipole

A : surface of the loop, N : cable turns

Magnetic dipole moment : $\vec{\mu} = NI\vec{A}$ $[Am^2]$

Always $\perp$ to plane of loop

$\vec{\tau} = \vec{\mu} \times \vec{B}$

Potential energy : $U = -\vec{\mu} \cdot \vec{B}$

Second Maxwell (no magnetic charges) :

$\nabla \cdot B = 0$

$\Phi_B \equiv \oint \vec{B}\, d\vec{S} = 0$

4.2 Ampere's law

Circulation of magnetic field : $\oint_{\partial\Sigma} B \cdot dl = 4\pi k_M \int_\Sigma \vec{j} \cdot d\vec{S}$

Fundamental velocity :

$c = \sqrt{\frac{k_E}{k_M}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} \approx 3 \times 10^8$ $[ms^{-1}]$

4-th Maxwell's (Ampère's law) :

$\nabla \times B = 4\pi k_M \vec{j} + \frac{1}{c^2} \frac{\partial\vec{E}}{\partial t}$

$\oint \vec{B} \cdot dl = \mu_0 I_{\text{enclosed}} + \frac{1}{c^2} \frac{\partial\Phi_E}{\partial t}$

5 Waves

In absence of charge ($\nabla \cdot E = 0$)

$\vec{E}$ and $\vec{B}$ satisfy the wave equation :

$\left(\frac{\partial^2}{c^2 \partial t^2} - \Delta\right) \vec{F} = 0$

Harmonic solution : simplest solution

$f = A\cos(kx \pm \omega t + \phi_0)$

amplitude A, wavelength $\lambda := \frac{2\pi}{k}$, period $T = \frac{2\pi}{\omega}$, angular frequency ω

$\omega = kc \equiv \frac{2\pi c}{\lambda}$, $\lambda = cT \equiv \frac{c}{f}$

In a harmonic wave : $\vec{k}$ is propagation direction :

$\vec{E} \perp \vec{B} \perp \vec{k}$, $\vec{k}$ directed along $\vec{E} \times \vec{B}$

EM waves : charge move with nonzero acceleration, accelerating charges $\Longleftrightarrow$ alternating electric current

Interference : 2 waves may amplify (cancel) each other if they oscillate together (out of phase, phase difference of 180)

Diffraction : each point of wavefront is source of spherical waves, may happen when light ray passes through a slit with width comparable to wavelength

6 Circuits

ρ : resistivity, l : length, S : cross section of wire

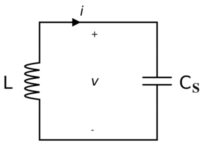
Resistance of material : $R = \rho\frac{l}{S}$

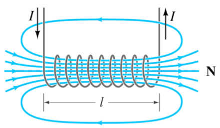
High temperature $\Longrightarrow$ large resistance

τ : time interval to next collision

Electron speed : $v_{\text{drift}} = \frac{eE}{m_e}\tau$

6.1 LC circuit





$\omega = \frac{1}{\sqrt{LC}}$, $I(t) = I_0\sin(\omega t)$

$U(t) = U_0\cos(\omega t)$, $L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$

Solenoid : $L = \mu_0 V n^2$

Inductance L : coefficient of proportionality between magnetic flux and current,

$\Phi_B = LI$ $[H]$ $[\Omega s]$ $[VsA^{-1}]$

Opposes changes in current (according to Faraday's law) : $U = -L\frac{dI}{dt}$

Capacitance C : coefficient of proportionally between accumulated change Q and potential difference $Q = CU$ $[F]$ $[AsV^{-1}]$

Leads to potential difference : $U = \frac{Q}{C}$

6.2 Kirchhoff

Junction rule : sum of incoming currents = sum of outgoing currents

Loop rule : directed sum of potential differences (voltages) on closed loop is 0

Resistors :

series : $R_S = R_1 + R_2$

parallel : $R_P = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

Inductance :

series : $L_S = L_1 + L_2$

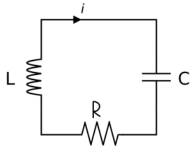
parallel : $L_P = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}$

Capacitors :

series : $C_S = \frac{1}{\frac{1}{C_1} \frac{1}{C_2}}$

parallel : $C_P = C_1 + C_2$

6.3 RLC circuit



$L\frac{dI}{dt} + RI + \frac{Q}{C} = 0$

$x(t) = Q(t)$, $w_0 = \frac{1}{\sqrt{LC}}$, $\gamma = \frac{R}{2L}$

$Q(t) = Q_0 \cdot e^{-\gamma t} \cdot \cos\sqrt{w_0^2 - \gamma^2 t}$

7 Cheat

$(|\varepsilon|)\ U = RI$

$R = \rho\frac{l}{S}$

$U = \frac{Q}{C}$

$F = QE$

$E = k\int_V \frac{1}{r^2} dq$

$B = \frac{\mu_0 I}{4\pi}(\hat{I} \times \hat{r}) \int_V \frac{1}{r^2}\, dl$

$F = k\frac{qq}{r^2}$, $k = \frac{1}{4\pi\epsilon_0}$

$U = k\frac{qq}{r}$

$|\varepsilon| = -\frac{d\Phi_B}{dt}$, $\Phi_B = BLx$

self inductance (L $[H]$) : $\Phi_B = \mu_0 n^2 lAI = LI$

solénoïde : $\Phi_B = BSn$, $B = \mu_0 N\frac{I}{l}$

$|F| = BIL$

Dipole : $E = \frac{kp}{l^3}$

Gauss : $\oint_S E \cdot dS = ES = \frac{Q_{\text{int}}}{\epsilon_0}$

Ampère static : $\int_{\partial\Sigma} B\, dl = BP = \mu_0 I_{\text{int}}$

Ampère dynamic : $= \mu_0\epsilon_0 V\frac{dE}{dt}$

Magnetic dipole moment : $m = nIA$, $U = -mB$

Torque magnetic : $\tau = \vec{m} \times \vec{B}$

$W_{ext} = \Delta U$, $W = -\Delta U$

Force per unit length : $\frac{F}{l} = 2K_m\frac{I_1 I_2}{r}$

$\Delta V = -\frac{d\Phi_B}{dt}$

Interference pattern : $\sin(\theta_n, \text{max}) = n\frac{\lambda}{d}$, $\sin(\theta_n, \text{min}) = (n + \frac{1}{2})\frac{\lambda}{d}$

Spacing btw. neighboring pts : $x = l\tan\theta$