Analyse 2 - CMS - Résumé

1er semestre

Angle: ↻ $-$, ↺ $+$, $L = \alpha \cdot r$, $A = \frac{1}{2} \cdot \alpha \cdot r^2$, ω: vitesse angulaire $[rad \cdot s^{-1}]$, $\alpha(t) = \omega \cdot t$,
$v = \frac{\omega}{2\pi}$, $T = \frac{1}{v} = \frac{2\pi}{\omega}$ (↳ période), $1 \text{ tour} = 360° = 2\pi$, $\alpha \cdot \frac{180}{\pi}$ [deg.], $\alpha \cdot \frac{\pi}{180}$ [rad.],

Cercle trigo.: $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$, $\cot\alpha = \frac{1}{\tan\alpha}$

	0°	30°	45°	60°	90°
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
tan	0	$\sqrt{3}/3$	1	$\sqrt{3}$	$\emptyset$
cot	$\emptyset$	$\sqrt{3}$	1	$\sqrt{3}/3$	0

$\cos\alpha: \mathbb{R} \to [-1;1]$, $\sin\alpha: \mathbb{R} \to [-1,1]$,
$\cos\alpha = \cos(-\alpha)$ (paire), $-\sin\alpha = \sin(-\alpha)$ (impaire),
$-\tan\alpha = \tan(-\alpha)$ (impaire), $-\cot\alpha = \cot(-\alpha)$ (impaire),
CAH SOH TOA, $\sin(\pi/2 - \alpha) = \cos\alpha$,
$\cos(\pi/2 - \alpha) = \sin\alpha$, $\tan(\pi/2 - \alpha) = \cot\alpha$,
$\cot(\pi/2 - \alpha) = \tan\alpha$,

$\cos(\pi - \alpha) = -\cos\alpha$, $\sin(\pi - \alpha) = \sin\alpha$, $\tan(\pi - \alpha) = -\tan\alpha$, $\cot(\pi - \alpha) = -\cot\alpha$

Réciproques: $\sin\alpha = x$ $x, \alpha \in \mathbb{R}$ $S = \{\arcsin x + 2k\pi, \pi - \arcsin x + 2k\pi\}$, $(k \in \mathbb{Z})$,
$\cos\alpha = x$ $x, \alpha \in \mathbb{R}$ $S = \{\arccos x + 2k\pi, -\arccos x + 2k\pi\}$, $\tan\alpha = x$ $S = \{\arctan c + k\pi\}$,
$\cot\alpha = x$ $S = \{\text{arccot}\, c + k \cdot \pi\}$, $\arcsin: [-1;1] \to [-\frac{\pi}{2}; \frac{\pi}{2}]$, $\arccos: [-1;1] \to [0;\pi]$,
$\arctan: \mathbb{R} \to]-\pi/2; \pi/2[$, $\text{arccot}: \mathbb{R} \to]0;\pi[$, $\sin(\arcsin(x)) = x$ $\cos(\arccos(x)) = x$ $\forall x \in [-1;1]$
$\tan(\arctan(y)) = y$ $\cot(\text{arccot}(y)) = y$ $\forall y \in \mathbb{R}$, $\arcsin(\sin(\alpha)) = \alpha$ $\forall \alpha \in [-\frac{\pi}{2}; \frac{\pi}{2}]$,
$\arccos(\cos(\alpha)) = \alpha$ $\forall \alpha \in [0;\pi]$, $\arctan(\tan(\alpha)) = \alpha$ $\forall \alpha \in]-\frac{\pi}{2}; \frac{\pi}{2}[$, $\text{arccot}(\cot(\alpha)) = \alpha$ $\forall \alpha \in]0;\pi[$

Formules: $\cos^2\alpha + \sin^2\alpha = 1$, $\frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$, $\frac{1}{\sin^2\alpha} = 1 + \cot^2\alpha$, $\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$,
$\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$, $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$, $\cot(\alpha \pm \beta) = \frac{-(1 \mp \cot\alpha \cdot \cot\beta)}{\cot\alpha \pm \cot\beta}$,
$\sin(2\alpha) = 2\sin\alpha \cdot \cos\alpha$, $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$, $\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$, $\cot(2\alpha) = \frac{\cot^2\alpha - 1}{2\cot\alpha}$,
$\cos^2(\frac{\alpha}{2}) = \frac{1 + \cos\alpha}{2}$, $\sin^2(\frac{\alpha}{2}) = \frac{1 - \cos\alpha}{2}$, $\tan^2(\frac{\alpha}{2}) = \frac{1 - \cos\alpha}{1 + \cos\alpha}$, $\cot^2(\frac{\alpha}{2}) = \frac{1 + \cos\alpha}{1 - \cos\alpha}$,
$\sin\alpha \pm \sin\beta = 2\sin(\frac{\alpha \pm \beta}{2}) \cdot \cos(\frac{\alpha \mp \beta}{2})$, $\cos\alpha + \cos\beta = 2\cos(\frac{\alpha+\beta}{2}) \cdot \cos(\frac{\alpha-\beta}{2})$, $\cos\alpha - \cos\beta = -2\sin(\frac{\alpha+\beta}{2}) \cdot \cos(\frac{\alpha-\beta}{2})$,
$\sin\alpha \cdot \cos\beta = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$, $\cos\alpha \cdot \cos\beta = \frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2}$, $\sin\alpha \cdot \sin\beta = -\frac{\cos(\alpha+\beta) - \cos(\alpha-\beta)}{2}$

Harmoniques: $c = \sqrt{A^2 + B^2}$, $\cos\varphi = \frac{A}{C}$, $\sin\varphi = \frac{B}{C}$, $\tan\varphi = \frac{B}{A}$,
$A \cdot \sin(\omega t + \varphi) + B \cdot \cos(\omega t + \varphi) = C \cdot \cos(\omega t - \varphi)$ (par ex.)

Bioche: $R(\sin x, \cos x)$, $(x \leftrightarrow -x)$ $z := \cos x$ $\sin^2 x = 1 - z^2$, $(x \leftrightarrow \pi - x)$ $z := \sin x$ $\cos^2 x = 1 - z^2$

$(x \leftrightarrow \pi + x)$ $z := \tan x$ $\quad \sin^2 x = \frac{z^2}{1+z^2}$ $\quad \cos^2 x = \frac{1}{1+z^2}$ $\quad$ ⚠ tester $x = \frac{\pi}{2} + k\pi$, (sinon) $z := \tan(\frac{x}{2})$

$\tan x = \frac{2z}{1-z^2}$ $\quad \cos x = \frac{1-z^2}{1+z^2}$ $\quad \sin x = \frac{2z}{1+z^2}$ $\quad$ ⚠ tester $x = \pi + 2k\pi$ $\quad$ Sol: $x = 2\arctan(z_i) + 2k\pi$

Triangles: $\alpha + \beta + \gamma = \pi$, $\alpha \geq \beta \geq \gamma \Leftrightarrow a \geq b \geq c$ $\circlearrowleft$, $a < b + c$ $\circlearrowleft$, $a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$ $\circlearrowleft$,

$A = \frac{1}{2} a \cdot b \cdot \sin\gamma$ $\circlearrowleft$ $\Rightarrow \frac{2A}{a \cdot b \cdot c} = \frac{\sin\gamma}{c} = \frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$, $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2r$ (cercle circonscrit)

Dérivabilité: $\sin' x = \cos x$, $\cos' x = -\sin x$, $\tan' x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$, $\cot' x = -1 - \cot^2 x = -\frac{1}{\sin^2 x}$,

$\arcsin' x = \frac{1}{\sqrt{1-x^2}}$, $\arccos' x = -\frac{1}{\sqrt{1-x^2}}$, $\arctan' x = \frac{1}{1+x^2}$, $\text{arccot}' x = -\frac{1}{1+x^2}$

Logarithme: $(f(t) = \frac{1}{t})$, $\ln : \mathbb{R}_+^* \longrightarrow \mathbb{R}$, $\ln x = \int_1^x \frac{dt}{t}$, $\ln(1) = 0$, $\ln(\frac{1}{x}) = -\ln x$, $\ln(x_1 \cdot x_2) = \ln x_1 + \ln x_2$,

$\ln(\frac{x_1}{x_2}) = \ln x_1 - \ln x_2$, $\ln(x^y) = y \ln x$ $(y \in \mathbb{R}, x_1, x_2, x > 0)$, ln: strictement croissante / continue / dérivable,

$\ln' x = \frac{1}{x}$, $\ln(e) = 1$, $e = \sum_{k=0}^{\infty} \frac{1}{k!} = \lim_{n \to \infty} (1 + \frac{1}{n})^n$

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2ème semestre

Log. et exp.: $\log_b: \mathbb{R}_+^* \to \mathbb{R}$ $\log_b(a)=x \Leftrightarrow \exp_b(x)=a$ $\forall 1\neq b>0$, $\log_b(a)=\frac{\ln(a)}{\ln(b)}$, $\frac{d}{dx}\log_b(x)=\frac{1}{x\cdot\ln(b)}$, $\exp_b: \mathbb{R}\to\mathbb{R}_+^*$ $\exp_b(x)=\exp(\ln(b)\cdot x)=e^{x\cdot\ln b}=b^x$ $\forall b>0$, $\frac{d}{dx}a^x=a^x\cdot\ln(a)$, $f^g(x)=\exp(\ln(f(x))\cdot g(x))=f(x)^{g(x)}$, $\exp(-x)=\frac{1}{\exp(x)}$, $\exp(x+y)=\exp(x)\cdot\exp(y)$, exp. est croissante et continue, $\lim_{x\to-\infty}\exp(x)=0$, $\exp(-x)\cdot\exp(x)=1$, $\exp_b(0)=1$, $b^{x_1+x_2}=b^{x_1}\cdot b^{x_2}$, $b^{x_1-x_2}=\frac{b^{x_1}}{b^{x_2}}$, $\exp_b(x)=\exp_{\frac{1}{b}}(-x)$, $\exp_b(x)\cdot\exp_c(x)=\exp_{b\cdot c}(x)$, $(a^b)^c=a^{bc}$, $\ln(a^x)=x\cdot\ln(a)$, $\log_b(1)=0$, $\log_b(x)=-\log_{\frac{1}{b}}(x)$, $\log_a(b^c)=c\log_a(b)$, $\frac{d}{dx}f^{-1}(x)=\frac{1}{f'(f^{-1}(x))}$

Fonctions hyperboliques: paire: $f(-x)=f(x)$, impaire: $f(-x)=-f(x)$, $\cosh(x)=\frac{e^x+e^{-x}}{2}$, $\cosh^{-1}(x)=\ln(x+\sqrt{x^2-1})$ $(\mathbb{R}_+\to[1;+\infty[)$, $\sinh(x)=\frac{e^x-e^{-x}}{2}$ (bijectif), $\sinh^{-1}(x)=\ln(x+\sqrt{x^2+1})$, cosh paire / sinh impaire, $\cosh(0)=1$ / $\sinh(0)=0$, $\cosh>\sinh$, $\cosh^2(x)-\sinh^2(x)=1$, $\sinh'(x)=\cosh(x)$ / $\cosh'(x)=\sinh(x)$, $\tanh: \mathbb{R}\to\mathbb{R}$ $x\mapsto\frac{\sinh(x)}{\cosh(x)}$, $\coth: \mathbb{R}^*\to\mathbb{R}$ $x\mapsto\frac{\cosh(x)}{\sinh(x)}$, tanh et coth impaires, $\tanh(0)=0$, $\coth>\tanh$, $\tanh^2(x)=1-\frac{1}{\cosh^2(x)}$, $\tanh'(x)=\frac{1}{\cosh^2(x)}$, $\coth'(x)=-\frac{1}{\sinh^2(x)}$, $\tanh^{-1}(x)=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ $(x\in]-1;1[)$, $\coth^{-1}(x)=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ $(x\notin[-1;1])$

Complexes: $\mathbb{C}=\{xI_2+yi : x,y\in\mathbb{R}\}$ $i=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$, $z=x+iy$ x: partie réelle y: partie complexe, $\bar{z}=x-iy$, $|z|=|\bar{z}|=\sqrt{x^2+y^2}=\sqrt{\bar{z}\cdot z}$, $z\neq0 \Leftrightarrow |z|\neq0$, $Re(z)=\frac{1}{2}(z+\bar{z})$, $|z\cdot z'|=|z|\cdot|z'|$, $\overline{zz'}=\bar{z}\cdot\bar{z}'$, $z^{-1}=\frac{1}{|z|^2}\bar{z}$, $|z^{-1}|=\frac{1}{|z|}$, bijection avec $\mathbb{R}^2$: $z+z' \triangleq \begin{pmatrix}x+x'\\y+y'\end{pmatrix}$ / $z\cdot z'=\begin{pmatrix}x&-y\\y&x\end{pmatrix}\begin{pmatrix}x'\\y'\end{pmatrix}$, polaire: $z=x+iy=[|z|;\theta]$, $z=|z|(\cos\theta+i\cdot\sin\theta)$, $\theta=\arg z$, $\arg\bar{z}=-\arg z \mod 2\pi$, $\arg(-z)=\pi+\arg z \mod 2\pi$, $\arg z=\begin{cases}2\arctan\left(\frac{y}{|z|+x}\right) & \text{si } z\neq-|z|\\ \pi & \text{si } z=-|z|\end{cases}$, $zz'=[r\cdot s;\theta+\varphi \mod 2\pi]$, $z^{-1}=\left[\frac{1}{|z|};-\arg z \mod 2\pi\right]$, $|z|=r$, $[r^n,n\theta]=[1,0] \Leftrightarrow \begin{cases}r^n=1 & r>0\\ n\theta=0 \mod 2\pi & \theta\in]-\pi;\pi[\end{cases}$

Polynômes: $P(x)=\sum_{k=0}^{\deg P}a_k x^k$, $\deg(P+Q)\leq\max(\deg P,\deg Q)$, $\deg(P\cdot Q)=\deg P+\deg Q$, $P=QM+R$ $\deg R<\deg Q$, idéal: $\forall P,Q\in I: P+Q\in I$ et $\forall P\in I$ $\forall Q\in\mathbb{K}[X]: P\times Q\in I$, $M_{P,Q}=\{AP+BQ \mid A,B\in\mathbb{K}[X]\}=PGDC(P,Q)\cdot\mathbb{K}[X]$, P est irréductible ssi.: $\deg P=1$ ou $\deg P=2$ $\Delta_P<0$, éléments simples: $\frac{P(x)}{Q(x)}=M(x)+\frac{R(x)}{Q(x)}$ si $\deg P\geq\deg Q$ et on factorise $Q(x)$ pour obtenir par ex.: $\frac{R(x)}{Q(x)}=\frac{A_1}{x-\alpha}+\frac{A_2}{(x-\alpha)^2}+\frac{B_1x+C_1}{x^2+2ax+b}$ $\Rightarrow$ identification ou évaluation, viète: (deg.3) $a_0=-a_3(r_1\cdot r_2\cdot r_3)$ / $a_1=a_3(r_1r_2+r_2r_3+r_1r_3)$ / $a_2=-a_3(r_1+r_2+r_3)$ / $\rightarrow a_3x^3+a_2x^2+a_1x+a_0=0$

Application dérivées: dévelopement limité: $DL^{(n)}_{f,x_0}(x)=\sum_{k=0}^{n}\frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$, EDOL1h: $y'+py=0 \rightarrow y(x)=\lambda\exp\left(-\int_{x_0}^{x}p(s)\,ds\right)=\lambda\varphi_h(x)$ $y(x_0)=\lambda$, EDOL1: $y'+py=q$

$\rightarrow y(x) = \varphi_p(x) + \lambda \varphi_h(x)$ où $\varphi_h(x) = \exp\left(-\int_{x_0}^{x} p(t)\,dt\right)$: il faut deviner $\varphi_p(x)$: p, q constantes $\rightarrow \varphi_p(x) = \frac{q}{p}$ / $p, q \in \mathbb{R}[x]$ $\varphi_p(x) \in \mathbb{R}[x]$ $\deg \varphi_p = \deg q - \deg p$ / $q = A\cos x + B\sin x \rightarrow y = A\cos x + B\sin x$ et $y' = A\sin x + B\cos x$ addilionner y et y' regarder les coef. de l'EDOL / $\varphi_p(x) = \left(\int_{x_0}^{x} \frac{q(t)}{\varphi_h(t)}\,dt\right)\varphi_h(x)$